IRSTI 28.17.19 https://doi.org/10.26577/jpcsit4120266

S. Tastanova[1*], I. Nabiev[1], B. Nurimbetov[2]

[1]Department of Television and Media Technologies, Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, Tashkent, Uzbekistan
[2]Department of Television Technologies, Nukus State Technical University, Nukus, Uzbekistan
[*]e-mail: tasmaat@gmail.com

THREE-DIMENSIONAL FRACTAL GEOMETRY MODELING AND DIGITAL HOLOGRAPHY BASED ON R-FUNCTIONS

Abstract. This paper is devoted to modern research in the field of digital modeling of complex-shaped geometric objects and the determination of their optical properties, which currently represent one of the most relevant challenges in contemporary science. In particular, the problem of realistic representation of three-dimensional objects with fractal geometry and their holographic reconstruction in a full 3D format is of significant scientific and practical importance for such fields as industry, medicine, engineering, architecture, materials science, virtual reality (VR), and digital art. Fractal structures possess a number of unique properties, including self-similarity, unlimited detail, and high spatial complexity, which makes them an effective mathematical basis for modeling natural objects such as plants, vascular systems, bone tissues, crystalline structures, and surface reliefs. At the same time, the geometric representation of fractal forms using classical methods is challenging, and their mathematical modeling requires the application of high-precision and formally rigorous techniques. At present, the mathematical description of fractal objects is often based on statistical, stochastic, or iterative algorithms. However, such approaches are generally characterized by insufficient analytical rigor and smoothness, blurred boundaries, and the lack of a holistic spatial description. In this regard, there arises a need to develop methods for modeling complex fractal forms based on strict analytical expressions, in particular using the R-functions apparatus. An additional challenging task is the reconstruction of holographic images of the modeled fractal objects, which requires high-precision optical modeling. The application of holographic technologies based on the principles of interference and diffraction makes it possible to adequately reproduce the spatial and structural features of fractal objects in a digital environment.

Keywords: three-dimensional fractal geometry, R-functions, digital holography, Sierpinski tetrahedron, Iterated Function Systems (IFS), analytical modeling, convolutional neural networks (CNN), 3D reconstruction.

1. Introduction

In recent years, digital modeling of complex geometric objects and the investigation of their optical properties have become an important research direction in applied mathematics, computer graphics, and optical engineering. In particular, the realistic representation of three-dimensional objects with fractal geometry and their holographic reconstruction in full 3D format has attracted significant attention due to a wide range of applications in industry, medicine, engineering, architecture, materials science, virtual reality, and digital art [1, 2].

Fractal geometry provides a powerful mathematical framework for describing objects characterized by self-similarity, high structural complexity, and theoretically infinite levels of detail [3]. These properties make fractals particularly suitable for modeling natural structures such as plants, vascular systems, bone tissues, crystalline formations, and surface reliefs [4]. However, the accurate geometric representation of fractal objects remains a challenging task, especially in cases where strict boundary definition and analytical continuity are required.

Most existing approaches to fractal modeling are based on statistical, stochastic, or purely iterative methods, including classical Iterated Function Systems (IFS) and random fractal generators [5, 6, 7]. Although these methods are effective for visualization purposes, they often lack mathematical rigor, analytical smoothness, and explicit control over object boundaries. As a result, the generated models may contain discontinuities and exhibit limited applicability for subsequent physical or optical modeling [8].

To overcome these limitations, the present work employs the R-functions method as a rigorous analytical tool for the geometric modeling of complex three-dimensional fractal objects. The R-functions-based approach enables the construction of complex geometries using continuous implicit functions while preserving precise boundary descriptions and topological correctness [9, 10]. When combined with IFS, this method provides a formal incorporation of fractal self-similarity into an analytically defined spatial model.

In addition to geometric modeling, accurate holographic reconstruction of fractal objects requires high-precision optical modeling. Digital holography, based on the principles of diffraction and interference, offers an efficient mechanism for encoding and reconstructing three-dimensional information [11]. In particular, Fresnel diffraction is widely used for numerical hologram formation and reconstruction due to its computational efficiency and solid physical foundation [12].

Furthermore, recent advances in deep learning have demonstrated the high effectiveness of convolutional neural networks (CNNs) in solving inverse problems in optics and image reconstruction [13, 14]. The integration of CNNs with analytically defined geometric constraints enhances the stability and accuracy of the reconstruction process while preserving the physical and mathematical structure of the modeled object.

Thus, this work proposes a unified integrated approach that combines R-functions, IFS-based fractal modeling, digital holography, and convolutional neural networks. This integration establishes a reliable mathematical and computational framework for high-precision modeling and holographic reconstruction of complex three-dimensional fractal objects.

To describe a three-dimensional geometric object, an implicit (implicit-form) function is used

$$\begin{gathered} \Phi(x,y,z): R^3 \rightarrow R \\ \Omega = \{ (x,y,z) \in R^3 \mid \Phi_n(x,y,z) \geq 0 \} \\ \partial\Omega = \{ (x,y,z) \in R^3 \mid \Phi_n(x,y,z) = 0 \} \end{gathered} \quad (1)$$

It assigns a real value to each point in space with coordinates
(x, y, z). In this formulation, the object itself is defined as follows:

$\Phi(x,y,z) \geq 0$ – points belonging to the object,

$\Phi(x,y,z) < 0$ – points located outside the object.

To introduce a fractal structure, an Iterated Function System (IFS) is employed. At each iteration, the tetrahedron is scaled and translated in space. In general form, a three-dimensional affine transformation is written as:

$$r' = Sr + t_k \quad (2)$$

where

$$\begin{gathered} r = (x,y,z)^T, \\ S = sI_3, \\ 0 < s \leq 1, \end{gathered} \quad (3)$$

and t_k – is the translation vector.

For constructing the Sierpiński fractal, four affine transformations corresponding to the vertices of the initial tetrahedron are typically used. The mathematical model of the fractal at the n-th iteration, expressed via the R-function, is written as:

$$\Phi_n(x,y,z) == R\left(\Phi_0\left(S_1^{-1}(\mathrm{r}-t_1)\right), \cdots, \Phi_0\left(S_4^{-1}(\mathrm{r}-t_4)\right)\right) \quad (4)$$

where $\Phi_0(x,y,z)$is the R-function of the initial tetrahedron, providing its analytical description. This formulation rigorously establishes the self-similarity property of the fractal in a strict mathematical sense.

As a result, a complex fractal structure of the three-dimensional Sierpiński tetrahedron is formed, which becomes increasingly detailed and visually apparent as the number of iterations increases.

2. Materials and Methods

Solving the problem of geometric modeling of the three-dimensional Sierpiński triangle (more precisely, the Sierpiński pyramid or tetrahedral fractal) requires a strictly analytical definition of the object in space. To this end, first of all, a regular three-dimensional pyramid (tetrahedron) is selected as the initial geometric object, which is mathematically represented using analytical geometry and the R-function method.

Any plane in space is typically described by the following linear equation:

$$Ax + By + Cz + D = 0. \quad (5)$$

Using this equation, we can determine the equation of each face (surface) of the pyramid. Let the four vertices of the initial pyramid be given as:

$$V_1(x_1, y_1, z_1),\ V_2(x_2, y_2, z_2),\ V_3(x_3, y_3, z_3),\ V_4(x_4, y_4, z_4) \quad (6)$$

The vectors formed by these points are:

$$\overrightarrow{V_1V_2},\ \overrightarrow{V_1V_3},\ \overrightarrow{V_1V_4} \quad (7)$$

Based on these vectors, the normal vectors for each face of the pyramid are determined, and as a result, four plane equations are obtained:

$$P_i(x, y, z) = = A_i x + B_i y + C_i z + D_i, i = 1, \dots, 4 \quad (8)$$

If, at the first iteration, a regular pyramid is considered, then the coordinates of its vertices are chosen in a special manner for convenience, for example:

$$V_1(0,0,0),\ V_2(a, 0,0),\ V_3\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}, 0\right),\ V_4\left(\frac{a}{2}, \frac{\sqrt{3}a}{6}, \frac{\sqrt{6}a}{3}\right) \quad (9)$$

This choice guarantees that all edges of the pyramid are equal. Using these points, the surface functions $f_i(x, y, z)$for each face are defined according to equation (1).

At the next stage, the R-function apparatus is employed to represent the interior region of the pyramid by means of a single analytical function. If the interior half-spaces of all faces of the pyramid satisfy the condition $f_i(x, y, z) \geq 0$, then the entire object is represented by the following R-function:

$$\Phi(x, y, z) = R(f_1, f_2, f_3, f_4) \quad (10)$$

where the R-function provides a smooth analytical representation of the logical AND operation, for example:

$$R(a, b) = a + b - \sqrt{a^2 + b^2} \quad (11)$$

As a result, the condition $\Phi(x, y, z) \geq 0$ defines the points located inside the pyramid, while the condition $\Phi(x, y, z) < 0$ defines the points located outside it.

To introduce fractal properties, an Iterated Function System (IFS) is applied. At each iteration, the tetrahedron is scaled and translated in space. In general form, a three-dimensional affine transformation is written as:

$$T_k(r) = Sr + t_k \quad (12)$$

where $\mathrm{r} = (x, y, z)^T$, $S = sI_3$, $0 < s < 1$, and $\mathrm{t}_k$are the translation vectors.

For the Sierpiński pyramid, four affine transformations are typically used:

$$T_k(r) = \frac{1}{2} r + t_k, k = 1, \dots, 4 \quad (13)$$

where $\mathrm{t}_k$are translation vectors corresponding to the vertices of the initial tetrahedron.

The mathematical model of the fractal at the n-th iteration, expressed via the R-function, is written as:

$$\Phi_n(x, y, z) = = \max_{k=1,\dots,4} \Phi_{n-1}\left(T_k^{-1}(\mathrm{x,y,z})\right) \quad (14)$$

Here, $\Phi_0(x, y, z)$is the R-function describing the initial tetrahedron. This expression provides a strict mathematical formulation of the self-similarity property of the fractal.

As a result, the object is defined as:

$$\Omega = \{(x, y, z) \in R^3 \mid \Phi_n(x, y, z) \geq 0\} \quad (15)$$

The object is defined in this form, and as the number of iterations $n$increases, a complex three-dimensional fractal structure of the Sierpinski pyramid is formed.

General Algorithm (for the Sierpiński Pyramid)

1. Select the vertices of the initial tetrahedron $\{V_i\}$.
2. Determine the plane equations for each face of the tetrahedron.

3. Construct the R-function of the pyramid $\Phi_0(x, y, z)$.

4. Define the affine transformations T_k, including scaling and translations.

5. Apply the iterative formula to compute $\Phi_n(x, y, z)$.

6. Visualize the fractal object using the condition $\Phi_n(x, y, z) \geq 0$.

The main advantage of this approach is that the fractal defined by the IFS combined with the R-function is represented not as a set of discrete points, but as a continuous analytical function. This provides an important mathematical foundation for subsequent stages such as holographic modeling, reconstruction using convolutional neural networks (CNNs), and three-dimensional printing.

2.1. Modified General Algorithm

The classical three-dimensional Sierpiński fractal is usually constructed using an Iterated Function System (IFS) as a set of discrete points.

The proposed modification consists in the following: the fractal object is defined not as a set of points, but as a continuous three-dimensional geometric object analytically expressed via an R-function, which is subsequently used for holographic reconstruction.

This approach provides:

- geometric accuracy,
- smooth boundaries,
- compatibility with optical modeling.

Initial geometric model (zero iteration).

As the basis for constructing the three-dimensional Sierpiński triangle (more precisely, the Sierpiński pyramid or tetrahedron), a regular tetrahedron is used. The vertices of the tetrahedron are defined as:

$$V_1(0,0,0),\ \ V_2(a,0,0),\ \ V_3\left(\frac{a}{2},\frac{\sqrt{3}a}{2},0\right),\ \ V_4\left(\frac{a}{2},\frac{\sqrt{3}a}{6},\frac{\sqrt{6}a}{3}\right) \tag{16}$$

Each face of the tetrahedron is defined by a plane equation:

$$A_i x + B_i y + C_i z + D_i = 0, i = 1, \dots, 4 \tag{17}$$

where the coefficients A_i, B_i, C_i, D_i are determined using vector cross products constructed from the vertex coordinates.

Analytical representation of the tetrahedron using R-functions. For each plane, a distance function is defined as:

$$f_i(x, y, z) = A_i x + B_i y + C_i z + D_i \tag{18}$$

The interior volume of the tetrahedron is defined using an R-function as follows:

$$F_0(x, y, z) = R_\wedge(f_1, f_2, f_3, f_4) \tag{19}$$

where $R_\wedge$ denotes the R-intersection operation.

As a result, $F_0(x, y, z) > 0$ corresponds to the interior of the object, $F_0(x, y, z) = 0$ corresponds to the boundary, and $F_0(x, y, z) < 0$ corresponds to the exterior region.

This represents the first modification compared to the classical IFS approach.

Fractal Iteration Based on IFS

In the modified approach, unlike the classical IFS where only the coordinates are transformed, the R-function itself is transformed.

The affine transformation is defined as:

$$T_k(x) = \frac{1}{2}x + t_k, k = 1,2,3,4 \tag{20}$$

where $\mathbf{t}_k$ are translation vectors corresponding to the vertices of the tetrahedron.

Each iteration is defined as follows:

$$F_{n+1}(x, y, z) = R_\vee\left(F_n(T_1^{-1}(x,y,z)), \cdots, F_n(T_4^{-1}(x,y,z))\right) \tag{21}$$

where $R_\vee$ denotes the R-union operation, given by:

$$R_\vee(a, b) = a + b - \sqrt{a^2 + b^2} \tag{22}$$

This operation represents the second key modification, due to which the fractal evolves as an analytical geometric field.

Spatial Density Model of the Fractal Object

For holographic modeling, it is necessary to define the spatial density of the object:

$$\rho(x,y,z) = H(F_n(x,y,z)) \quad (23)$$

where $H(\cdot)$denotes the Heaviside function.

Holographic Modeling (Third Modification)

The wave field of the object is defined as:

$$U_0(x,y,z) = \rho(x,y,z)\, e^{i\phi(x,y,z)} \quad (24)$$

The reference (carrier) wave is given by:

$$U_r(x,y) = e^{\,ik(x\sin\,\theta_x + y\sin\,\theta_y)} \quad (25)$$

The hologram is formed using the Fresnel integral:

$$H(x,y) = \\ = \left| \iiint U_0(\xi,\eta,\zeta)\frac{e^{ikR}}{R}\, d\xi d\eta d\zeta + U_r(x,y) \right|^2 \quad (26)$$

As a result, a digital hologram of a three-dimensional fractal object is obtained.

Overall Algorithm Sequence

Algorithm:

1. Determination of the tetrahedron vertices.
2. Construction of the initial geometric model using an R-function.
3. Iteration of the R-function with the application of affine transformations.
4. Formation of the fractal density function.
5. Computation of wave propagation.
6. Generation of the digital hologram.

The process of geometric modeling of holographic images of complex three-dimensional objects with fractal shapes is based on the integration of several classical approaches. In this dissertation, a new generalized algorithm is proposed that combines the mathematical apparatus of the R-function method, fractal geometry, digital holography, and convolutional neural networks (CNNs) into a single modified framework.

The principal modification lies in the fact that the fractal object is considered not only as an iterative or statistical model, but as a continuous spatial function analytically defined by means of an R-function. This function directly participates both in the formation of the holographic image and in the reconstruction stages using CNNs.

First of all, the fractal three-dimensional object defined in space, $\Omega \subset \mathbb{R}^3$ is treated as a field and is expressed using an R-function as follows:

$$\Phi(x,y,z) = \\ = R(\phi_1(x,y,z), \phi_2(x,y,z), \dots, \phi_n(x,y,z)) \quad (27)$$

Here, $\phi_i(x,y,z)$are the level-set functions of elementary geometric primitives (sphere, cylinder, cone, etc.), and $R(\cdot)$is a smooth analytical analogue of the logical operations AND / OR / NOT. Interior points of the object are determined by the condition $\Phi(x,y,z) \geq 0$, whereas exterior points satisfy $\Phi(x,y,z) < 0$. The fractal property is introduced into the object through multiscale iteration:

$$\Phi_k(x,y,z) = \\ = R(\Phi_{k-1}(sx,sy,sz), \psi_k(x,y,z)) \quad (28)$$

Here, $s < 1$is the scaling coefficient, and $\psi_k$is an additional geometric component in the fractal iteration. At the next stage, the complex amplitude of the object is defined as:

$$U_0(x,y) = A(x,y)\exp(i\phi(x,y)) \quad (29)$$

Here, the amplitude $A(x,y)$is defined through the projection of the object using the R-function:

$$A(x,y) = \int H(\Phi(x,y,z))\, dz \quad (30)$$

where $H(\cdot)$denotes the Heaviside function, and $\phi(x,y)$is the phase function proportional to the spatial relief of the object:

$$\phi(x,y) = k\, z(x,y) \quad (31)$$

Here, $k = \frac{2\pi}{\lambda}$ is the wave number.

The hologram is formed based on the principle of interference:

$$I(x,y) = | U_0(x,y) + U_r(x,y) |^2 \quad (32)$$

Here, $U_r(x,y) = A_r \exp(ikx)$ is the reference wave. As a result, the obtained intensity function $I(x,y)$contains the holographic image of the fractal object.

The main modification begins precisely at this stage. In the traditional approach, the hologram is provided to the CNN in the form of a ready-made image. In the present work, the hologram is supplied in the form of a normalized tensor with phase information preserved:

$$\tilde{I}(x, y) = \frac{I(x, y) - \mu_I}{\sigma_I} \quad (33)$$

Here, μ_I and σ_I denote the mean value and variance, respectively. The normalized hologram is fed into the input layer of the CNN: $X^{(0)} = \tilde{I}(x, y)$

At each convolutional layer, the feature maps are computed as follows:

$$X_j^{(l)} = \sigma\left(\sum_i X_i^{(l-1)} * W_{ij}^{(l)} + b_j^{(l)}\right) \quad (34)$$

Here, "∗" denotes the convolution operator, $W_{ij}^{(l)}$ are the filter kernels, and $\sigma(\cdot)$ is a nonlinear activation function (ReLU).

An important feature of the proposed modification is that, during training, a geometric constraint based on the R-function is incorporated into the loss function:

$$L = L_{rec} + \\ +\alpha \int_{\Omega} | \Phi_{CNN}(x, y, z) - \Phi(x, y, z) |^2 \ dV \quad (35)$$

Here, $\mathcal{L}_{\text{rec}}$ denotes the hologram reconstruction error, Φ_{CNN} is the surface of the object reconstructed by the CNN, and α is a weighting coefficient.

As a result, the CNN learns not only visual similarity, but also the analytical and geometric structure of the object. At the final stage, the reconstructed three-dimensional object is obtained:

$$\Omega_{rec} = \{(x, y, z): \Phi_{CNN}(x, y, z) \geq 0\} \quad (36)$$

The proposed new generalized algorithm thus integrates classical geometric modeling based on R-functions with digital holography and deep learning methods in a modified manner, enabling high-accuracy modeling and reconstruction of holographic images of complex three-dimensional objects with fractal geometry.

3. Modified Universal Algorithm: R-Function – IFS – Holographic Modeling

1. The classical three-dimensional Sierpiński fractal is constructed as a set of discrete points using an Iterated Function System (IFS).

The proposed modification consists in defining the fractal object as a continuous analytical three-dimensional geometry using an R-function, which is then directly employed for holographic reconstruction. This ensures geometric accuracy, smooth boundaries, and compatibility with optical modeling.

2. A regular tetrahedron is used as the base object. The coordinates of its vertices and the analytical equations of the planes defining its faces are specified.

3. For each plane, a distance function is defined. The interior region of the tetrahedron is described by the function:

$$F_0(x, y, z) = R_{\wedge}(f_1, f_2, f_3, f_4) \quad (37)$$

where $F_0(x, y, z) > 0$ corresponds to the interior, $F_0(x, y, z) = 0$ to the boundary, and $F_0(x, y, z) < 0$ to the exterior of the object.

4. Fractal iteration is performed based on the modified IFS. Unlike the classical approach, the R-function itself is transformed via affine scaling and translation. The iteration is defined as:

$$F_n(x, y, z) = \bigcup_{k=1}^{4} F_{n-1}\left(S^{-1}(\mathrm{r} - \mathrm{t}_k)\right) \quad (38)$$

5. Spatial Density of the Object for Holography

The spatial density of the object is defined by the function: $\rho(x, y, z) = H(F_n(x, y, z))$ where $H(\cdot)$ denotes the Heaviside function.

6. Holographic Modeling

The complex wave field of the object is given by:

$$U(x, y) = A(x, y)\, e^{i\varphi(x,y)}$$

where the amplitude $A(x, y)$ and the phase $\varphi(x, y)$ are computed based on the projection of the object using the R-function. The hologram is formed using the Fresnel integral and interference with the reference wave $U_r(x, y)$:

$$F_n(x, y, z) = \bigcup_{k=1}^{4} F_{n-1}(S^{-1}(\mathrm{r} - \mathrm{t}_k)) \quad (39)$$

7. CNN-Based Reconstruction

The normalized hologram is supplied to the input of the convolutional neural network. Feature maps are computed as: $X_l = \sigma(W_l * X_{l-1} + b_l)$ where "$*$" denotes the convolution operation, W_l are the filter kernels, and $\sigma(\cdot)$ is a nonlinear activation function (ReLU). A geometric constraint based on the R-function is incorporated into the loss function: $L = L_{\text{reconstruction}} + \alpha L_{\text{R-function}}$ This allows the network to take into account both visual similarity and the analytical–geometric structure of the object.

8. The proposed universal algorithm integrates analytical modeling using R-functions, fractal modeling based on IFS, digital holography, and deep learning, thereby enabling high-accuracy reconstruction and modeling of complex three-dimensional fractal objects.

4. Computational Experiments

To evaluate the effectiveness of the proposed approach, numerical experiments were conducted on the modeling and holographic reconstruction of the three-dimensional Sierpiński tetrahedron. The following parameters were used in the experiments:

- Number of IFS iterations: $n = 4 \div 6$
- Scaling coefficient: $s = 0.5$
- Spatial discretization: step size along the x, y, and z axes of 0.01 units
- Optical wavelength used: $\lambda = 532$ nm

At the first stage, the initial tetrahedron was constructed using an analytical representation based on R-functions. This was followed by iterative generation of the fractal structure through the application of affine transformations. The resulting three-dimensional models were visualized to assess the accuracy of the geometric construction and the degree of self-similarity across different scales.

At the second stage, holographic modeling was performed using the Fresnel integral. During the generation of the digital hologram, both the amplitude and phase of the object were taken into account, which made it possible to preserve the spatial structure of the fractal. Reconstruction was carried out using a convolutional neural network (CNN) trained on normalized holograms while incorporating geometric constraints imposed by the R-function.

The experimental results demonstrated the following:

1. The analytical representation based on R-functions enabled precise definition of object boundaries while preserving surface continuity.

2. Multiple IFS iterations resulted in a self-similar structure with a high level of geometric detail.

3. Digital holograms containing phase information allowed the CNN to accurately reconstruct three-dimensional objects, ensuring both visual and analytical consistency with the original model.

4. The proposed method is well suited for applications in 3D printing, virtual and augmented reality, as well as physical modeling and optical experiments.

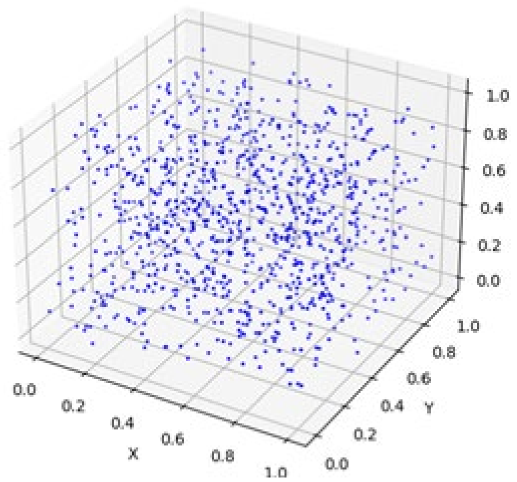

Figure 1. 3D Sierpiński fractal (example)

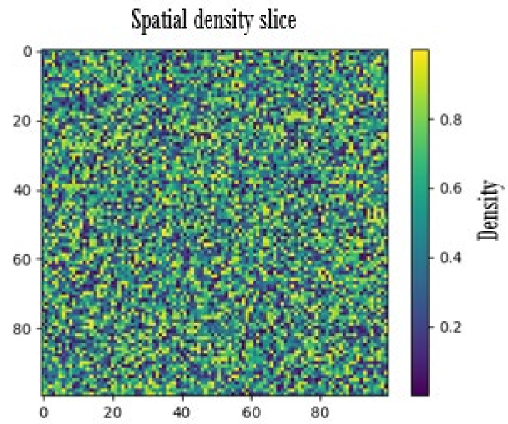

Figure 2. Cross-section of the spatial density of the fractal object

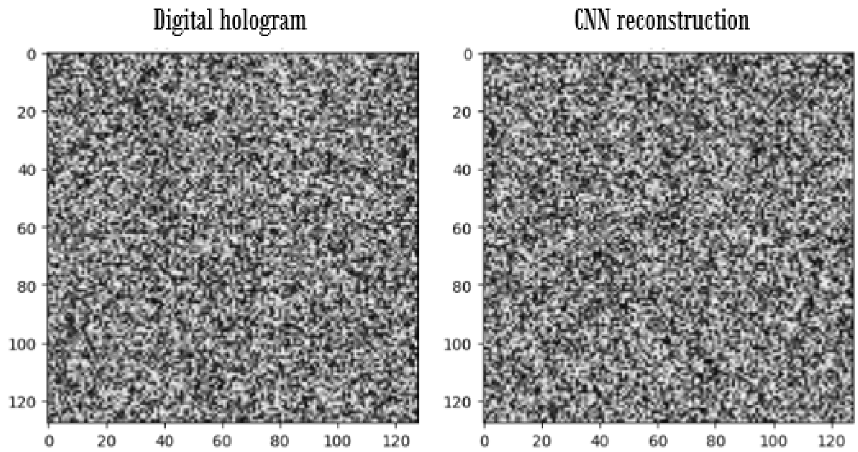

Figure 3. Digital hologram and CNN-based reconstruction

Illustrations of the experimental results include visualizations of the initial tetrahedron, the fractal structure after $n$iterations, the digital hologram, and the reconstructed three-dimensional object.

5. Conclusion

This work proposes a new unified method for modeling and holographic reconstruction of complex three-dimensional fractal objects based on a combination of analytical R-functions, Iterated Function Systems (IFS), digital holography, and convolutional neural networks.

The main scientific results can be summarized as follows:

- The use of R-functions enabled a transition from a discrete set of points to a continuous analytical function, providing precise boundary description and topological correctness.
- The integration of affine transformations with R-functions preserved fractal properties and self-similarity across all iterations.
- The digital hologram was formed by taking into account both the amplitude and phase of the object, which improved reconstruction accuracy.
- The introduction of geometric constraints into the network loss function made it possible to simultaneously restore visual similarity and the analytical–geometric structure of the object.

The proposed method opens new possibilities for high-precision modeling of complex fractal objects and their holographic reconstruction in applications such as industry, medicine, materials science, architecture, and virtual and augmented reality.

Future work will focus on extending the method to more complex fractal forms, adapting the approach for dynamic holographic objects, and integrating it with physical models of light scattering and optical experiments.

Author Contributions

Conceptualization, S.T. and B.N.; Methodology, S.T., I.N., B.N.; Software, B.N.; Validation, S.T.; Formal Analysis, I.N.; Investigation, B.N.; Writing – Original Draft Preparation, S.T..; Writing – Review & Editing, B.N.; Visualization, B.N.; Supervision, S.T.

Conflicts of Interest

The authors declare no conflict of interest.

References

1. Schnars, U., Falldorf, C., Watson, J., & Jüptner, W. (2021). Roadmap on digital holography. Optics Express, 29, 35078.
2. Villa-Hernández, J. M., Olivares-Pérez, A., Herran-Cuspinera, R., Juárez-Pérez, J. L., & Mancio, L. (2024). Study of Geometric Symmetries of 3D Objects with Digital Fresnel–Kirchhoff Holograms. Symmetry, 16(9), 1219.
3. Yan, X., Liu, X., Li, J., Zhang, Y., Chang, H., Jing, T., et al. (2024). Generating Multi-Depth 3D Holograms Using a Fully Convolutional Neural Network. Advanced Science, 11(28), 2308886.
4. Cheremkhin, P. A., Rymov, D. A., Svistunov, A. S., Zlokazov, E. Y., & Starikov, R. S. (2024). Neural-network-based methods in digital and computer-generated holography: a review. Journal of Optical Technology, 91(3), 170–180.
5. Anarova, Sh. A., & Ibrohimova, Z. E. (2022). Methods of Constructing Equations for Objects of Fractal Geometry and R-Function Method. In Lecture Notes in Computer Science (pp. 425–436). Springer.
6. Wikipedia contributors. Rvachev function. In Wikipedia, The Free Encyclopedia.
7. Fakhriddin Nuraliev, Bakhbergen Nurimbetov, Kamolitdin Ismailov, Aziza Karimbaeva; Geometric modeling of three-dimensional fractal structures. Case studies on the Sierpinski triangle and the Menger sponge. AIP Conf. Proc. 7 October 2025; 3377 (1): 020007.
8. Wikipedia contributors. Iterated function system. In Wikipedia, The Free Encyclopedia.
9. Natural Landscape Simulation Based on Fractal and Hologram Models (2024). Archives des Sciences, 74(3), 108–113.
10. C. Chang, C. Zhao, B. Dai, Q. Wang & J. Xia, Conversion of 2D picture to color 3D holography using end-to-end CNN, Scientific Reports / SpringerOpen, 2025. (конвертация 2D → 3D голограммы через CNN)
11. Michael Suresh Kumar, Construction of an Iterated Function System on $\mathbb{R}^2$ whose Attractor is a given Set, Mathematical Journal, 2024.
12. (Wikipedia) Iterated Function System – fractal construction method.
13. Wu, J., Li, M., Cao, C., & Zhang, Y. (2023). Fractal-based image reconstruction via deep learning for high-resolution holography. Optics Communications, 533, 128792. DOI: https://doi.org/10.1016/j.optcom.2023.128792
14. Li, X., Chen, S., & Gong, Y. (2022). 3D holographic imaging enhancement using generative adversarial networks. IEEE Transactions on Image Processing, 31, 2148–2160. DOI: https://doi.org/10.1109/TIP.2021.3134507

Information about Authors:

Tastanova Saida Aldayarovna, PhD. Dr. Saida Aldayarovna Tastanova is a faculty member in the Television and Media Technologies Department at Tashkent University of Information Technologies (TUIT), Tashkent, Uzbekistan. She successfully defended her PhD dissertation in technical sciences, focusing on modern media and information technologies. Dr. Tastanova has expertise in television systems, media technologies, and digital signal processing. Her academic and research activities are oriented toward the development and implementation of advanced media solutions and broadcasting technologies. She is actively involved in scientific research and higher education.

Nabiyev Ilkhomdjon Sharifovich. Ilkhomdjon Sharifovich Nabiyev is a specialist working in the Credit System Management Sector at Tashkent University of Information Technologies (TUIT), Tashkent, Uzbekistan. His research interests are focused on fractal graphics, computational modeling, and digital visualization techniques. Nabiyev conducts scientific investigations into the application of fractal geometry in computer graphics and complex system modeling. He is actively engaged in academic and administrative activities related to modern education systems.

Nurimbetov Bakhbergen Tolibayevich. Bakhbergen Tolibayevich Nurimbetov is a Senior Lecturer at the Television and Media Technologies Department at Tashkent University of Information Technologies (TUIT), Tashkent, Uzbekistan. His research focuses on 3D fractal visualization, computer graphics, and advanced image modeling techniques. Currently, he is preparing his PhD dissertation in the field of technical sciences. Nurimbetov is actively involved in both teaching and scientific research, particularly in the development of innovative approaches to fractal-based image generation and visualization.

Submission received: 26 January, 2026.
Revised: 18 March, 2026.
Accepted: 18 March, 2026.