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MODELING OF THE DEFORMED STATE OF MESH PLATES USING COMPLEX CONFIGURATION

Abstract: In this article, the deformed state processes of mesh plates with complex configurations are modeled mathematically. Specifically, a computational algorithm comprising of the R-function methods of V.L. Rvachev (RFM) and the Bubnov-Galerkin method is applied. The mathematical model describes the behavior of mesh plates under external loads by representing equilibrium equations in a Cartesian coordinate system. The solution structures are built using constructive RFM approaches, and discretization is carried out with the Bubnov-Galerkin technique. Computational experiments are conducted to determine the deformation characteristics of mesh plates with intricate geometries. The proposed approach significantly reduces the computational complexity and increases the accuracy of results when compared to conventional analytical methods. Furthermore, the algorithm enables numerical analysis of rhomboidal and hexagonal plate configurations under different boundary conditions. These results may be utilized in designing lightweight yet structurally efficient components in aerospace, civil, and mechanical engineering.

Keywords: mathematical modeling, stress-strain state, mesh plates with complex configurations, R-function method (RFM), Bubnov-Galerkin method, computational mechanics.

1. Introduction

Mesh shells and plates are common structural forms in different technological domains as well as the construction sector. In this sense, there has been a recent surge in interest in these structures both domestically in our country and elsewhere. The high industrial production of the primary structural components and a notable rise in the walls of their ready structures, the objectivity of the products, and the potential for broad unification of them not only for individual structures but also for buildings with different objects, loads, and operation patterns are the benefits of building structures. The theory of mesh systems was developed with contributions from well-known scientists including L.N. Lubo, B.A. Mironkov, G.I. Pshenichnov, V.K. Kabulov, T.Sh. Shirinkulov, T.Buriev, and K.S.Abdurashitov, among others. However, the configurations of these mesh plates have classical forms in many of the investigated engineering practice issues where the stress-strain condition of the plates is being studied. However, in engineering practice, complicated mesh plate designs present a problem for designers and design engineers to solve mathematically.

This makes the development of a computational algorithm and the software that goes along with it, as well as the computation of mesh plates with intricate configurations, highly pertinent.

2. Materials and methods

In the development of the theory and methods for calculating lattice shells and plates, significant contributions were made by such well-known scientists as G.I. Pshenichnikov, V.V. Kuznetsov, L.N. Lubo, B.A. Mironov, V.I. Volchenko, R.I.Khisamov, A.L. Filin, I.G. Tagiev, A.R.Rzhanitsyn, B.S. Volikov, V.V. Bolotin and others [1-11].

The work [6] presents a new, fairly accurate method for calculating the processes and stability of lattice cylindrical shells, which allows taking into account the main features of the structure under consideration. The results obtained earlier by the author, valid for shells with a square mesh, are

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generalized to the case of a shell with a rhombic lattice.

The theory of thin elastic lattice shells and plates, as well as its applications in technology, are presented in [7]. The main attention is paid to the issues of the theory of single-layer structures, mesh, which are rod systems.

Based on the hypotheses of the technical theory of thin-walled rods by V.Z. Vlasov, a system of differential equations of thin-walled composite rods with variable stiffness characteristics with elastically compliant shear connections was obtained by the displacement method [8]. It is shown that in this problem, the force method leads to a system of integro-differential equations if the shear forces in the seams are taken as the main unknowns. The solution of the problem in the case of static variability of stiffness characteristics is carried out by the perturbation method using Green's functions.

In [9], the effect of rigid gussets in nodes and the frequency and shape of oscillations of trusses with different numbers of panels are considered. Loads and masses are taken at the nodes. The effect of changing the stiffness of rods on the displacement of truss nodes is investigated.

In [10], theoretical studies of the strength of a two-support beam with a defect arbitrarily located along the length and height are presented. The beam is loaded with a concentrated force. Calculation formulas for determining the forces in the transverse ties are derived.

The author of the work [11] proposes to present an expression for the differential operator and derivatives under limiting and boundary conditions in finite differences for calculating a statically indeterminate beam of an asymmetric cross-section made of a material with different modulus. The corresponding system of algebraic equations is solved by the Seidel method, modified by the authors due to the presence of limiting conditions. An example of calculation is given.

In this research [12], vibration analysis of a thin elastic circular Aluminium plate has been investigated both experimentally and computationally. First six natural frequencies of thin Aluminium circular plate having diameter 220mm were calculated by performing modal analysis on ANSYS. In order to perform computational study, multizone quad/tri method was applied on the thin elastic circular plate to generate the quadrilateral mesh and simply supported boundary condition was applied. Mesh independence study was performed by varying the size of the mesh through sizing option and examining the natural frequency. To determine fundamental natural frequency experimentally, forced vibrations were induced into the plate with the help of system comprises of DC motor, steel wire and cam with the eccentricity of 8mm.

In this work [13], the conventional method of modeling and analyzing cold plates involves the use of empirical correlations to predict pressure drop for standard fin geometries used for various designs. A more reliable approach to pressure drop prediction is to perform a detailed CFD analysis of the cold plate design. Conventional CFD analysis of fluid flow through cold plates requires filling in details of the cold plates (passages and fin structures) as well as fluid volumes in the cold plates. For complex fin patterns produced by additive manufacturing, this can be very computationally expensive. A new analytical method for CFD analysis of the cold plate structure with only the captured fluid volume was developed, which significantly reduced the time required to mesh and solve complex fin structures in cold plate fluid passages.

Additive manufacturing (AM) technologies offer design freedom to create complex metal structures with significantly increased area-to-volume ratios with minimal time and cost. This allows creating a new generation of two-phase exchangers, thermosyphons and heat pipes. The pile design has been identified as the main parameter affecting the thermal performance of heat pipes and steam chambers. Traditionally, these rods were constructed by sintering metal powder or welding wire mesh welded to the inner wall of the heat pipe. This research paper [14] investigates the use of Laser Powder Bed Fusion (L-PBF) to construct AM microstructured rods that cannot be produced using conventional methods. Four pore configurations are produced using L-PBF: (i) body-centered cubic (BCC), (ii) face-centered cubic (FCC), (iii) regular cubic (SC), and (iv) Voronoi. The porosity of each configuration was varied by varying the strut thickness from 0.25 mm to 0.35 mm. The capillary performance of these rods was tested through a rate of rise experiment in which the sample is immersed vertically in a well of ethanol and the mass rate is recorded using an accurate laboratory scale.

In addition to the classical methods for analyzing the stress-strain behavior of lattice shells and plates, recent studies have explored the nonlinear deformation processes of anisotropic plates under complex fields. For instance, in [26], the authors developed a mathematical model based on the Hamilton-Ostrogradsky variational principle and Kirchhoff-Lyaw hypothesis to study the thermo-electro-magneto-elastic behavior of plates with intricate shapes. Their analysis integrated electromagnetic and thermal effects using Cauchy relations, Hooke's law, and Maxwell’s equations. Furthermore, a related investigation [27] addressed the independent solution of the magnetoelastic problem for thin compound-shaped isotropic plates. The study applied a variational approach to derive plate motion equations under electromagnetic fields, contributing to the theoretical base of compound geometries. These approaches reflect an increasing trend toward incorporating multi-physical interactions in plate modeling tasks.

From the review of studies it follows that among the considered mesh plates the classical configurations have practical significance. Even their algorithm has not been developed when the mesh plate configurations have complex shapes.

3. Results

Equilibrium equations for a mesh plate element in a Cartesian coordinate system relatively have the form [1-3]

$$\frac{\partial Q_1}{\partial x_1}+\frac{\partial Q_2}{\partial y}+Z=0 \tag{1}$$

where

$$Q_1=\frac{\partial H_2}{\partial y}-\frac{\partial M_1}{\partial x},\ Q_2=\frac{\partial H_1}{\partial x}-\frac{\partial M_2}{\partial y}$$

Here Z is the external load; the parameters are bending and torque; lateral forces.

Bending and torque moments, when the number of rods is equal to n (general case), are determined by the following formulas:

$$\left.\begin{array}{c} M_1=-(D_{11}\chi_1+D_{12}*\chi_2+2D_{16}*\tau)\\ M_2=-(D_{21}\chi_1+D_{12}*\chi_2+2D_{16}*\tau)\\ H_1=D_{61}*^{(i)}\chi_1+D_{62}*^{(i)}\chi_2+D_{66}*^{(i)}, i=1,2 \end{array}\right\} \tag{2}$$

in which

$$\left.\begin{array}{c} D_{11}^*=D_{11}+K_{11}, D_{12}^*=D_{12}-K_{12}, D_{16}^*=D_{16}-\frac{1}{2}K_{16},\\ D_{21}^*=D_{k1}-K_{11}, D_{22}^*=D_{22}+K_{22}, D_{26}^*=D_{26}+\frac{1}{2}K_{26},\\ D_{16}^{*(i)}=D_{61}+K_{61}^{(i)}, D_{26}^{*(i)}=D_{62}-K_{62}^{(i)},\\ D_{66}^{*(i)}=D_{66}-K_{66}^{(i)}, i=1,2, \end{array}\right\} \tag{3}$$

$$\left.\begin{array}{c} D_{11}=\sum_{i=1}^{n}I_iC_i^4, D_{12}=\sum_{i=1}^{n}I_iS_i^2C_i^2, D_{66}=2D_{12}\\ D_{12}=\sum_{i=1}^{n}I_iS_iC_i^3, D_{22}=\sum_{i=1}^{n}I_iS_i^4, D_{26}=\sum_{i=1}^{n}I_iS_i^3C_i, D_{ij}=D_{ij},\\ K_{61}^{(1)}=K_{62}^{(1)}=\sum_{i=1}^{n}C_iS_iC_i^3, K_{66}^{(1)}=\sum_{i=1}^{n}C_ic_i^2\cos 2\phi_i,\\ K_{61}^{(2)}=K_{62}^{(2)}=\sum_{i=1}^{n}C_iS_i^2C_i, K_{66}^{(2)}=\sum_{i=1}^{n}C_iS_i^2\cos 2\phi_i,\\ K_i=E_iF_i/a_i, I=F_iJ_{1i}/a_i, C_i=G_iJ_{3i/a_i}. \end{array}\right\} \tag{4}$$

Let's characterize the parameters provided in (4). I_i, C_i show the relationship between the rods' matching stiffness properties and the separation between their axes.; a_i – distance between the axes of adjacent rods; F_i – rod area; J_{1i}, J_{3i} – main, central moments of inertia; n – number of mesh bar families; φ_i – angle between axis α and the axis of the rod (measured from the axis α in the direction of the axis β); $S_i = \sin\varphi_i, C_i = \cos\varphi_i$,

$$x_1 = -\frac{\partial^2 W}{\partial x^2}, x_1 = -\frac{\partial^2 W}{\partial x^2}, \tau = -\frac{\partial^2 W}{\partial x \partial y}$$

Substituting (2) taking into account (3) into (1), we obtain the equilibrium equation of mesh plates with respect to deflection – W:

$$\begin{gathered}
(D_{11}+K_{11})\frac{\partial^2 W}{\partial x^2}+2(D_{61}+D_{16})\frac{\partial^4 W}{\partial x^3\,\partial y}+(3D_{66}+K_{16})\frac{\partial^4 W}{\partial x^2\,\partial y^2} \\
+2(D_{62}+K_{16})\frac{\partial^4 W}{\partial x\,\partial y^3}+(D_{22}+K_{11})\frac{\partial^4 W}{\partial y^4} \\
+[2\frac{\partial}{\partial x}(D_{11}+K_{11})\frac{\partial}{\partial y}(2D_{61}-K_{16})]\frac{\partial^3 W}{\partial x^3} \\
+[3\frac{\partial}{\partial x}(2D_{61}-K_{16})+\frac{\partial}{\partial y}(3D_{66}-K_1^0)]\frac{\partial^3 W}{\partial x^3\,\partial y} \\
+\frac{\partial}{\partial y}(3D_{66}+K_1^0)+3\frac{\partial}{\partial x}(2D_{62}+K_{16})]\frac{\partial^3 W}{\partial x\,\partial y^2} \\
+[\frac{\partial}{\partial y}(2D_{62}+K_{16})+2\frac{\partial}{\partial y}(D_{22}+K_{11})]\frac{\partial^3 W}{\partial y^3} \\
+[\frac{\partial^2}{\partial x^2}(D_{11}+K_{11})\frac{\partial}{\partial x\,\partial y}(2D_{61}-K_{16})+\frac{\partial^2}{\partial y^2}(2D_{61}+K_{16}) \\
+\frac{\partial}{\partial x\,\partial y}(2D_{66}+K_1^0)+\frac{\partial}{\partial y^2}(2D_{62}+K_{16})]\frac{\partial^2 W}{\partial \mathrm{x}} \\
+[\frac{\partial^2}{\partial x^2}(D_{12}-K_{11})+\frac{\partial^2}{\partial x\,\partial y}(2D_{62}+K_{16})+\frac{\partial^2}{\partial y^2}(D_{22}+K_{11})]\frac{\partial^2 W}{\partial y^2} \\
-Z=0
\end{gathered} \tag{5}$$

where

$$K_1^{(0)} = \sum_{i=1}^{h}(1-6j_i^2c_i^2)c_i\,, K_2^{(0)} = \sum_{i=1}^{h} C_i\,cos^2\,2\,\phi_i \tag{6}$$

Substituting (4) into (3), then the resulting relations into (2), we have:

$$\left.\begin{aligned}
M_1 &= \sum_{i=1}^{4} c_i\nabla_i\,(I_i c_i\nabla_i + C_i s_i\nabla_i)w, \\
M_2 &= \sum_{i=1}^{4} s_i\nabla_i\,(I_i s_i\nabla_i + C_i c_i\nabla_i)w, \\
H_1 &= -\sum_{i=1}^{4} c_i\nabla_i\,(I_i c_i\nabla_i - C_i c_i\nabla_i)w, \\
H_2 &= -\sum_{i=1}^{4} s_i\nabla_i\,(I_i c_i\nabla_i - C_i c_i\nabla_i)w.
\end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} Q_1 &= -\sum_{i=1}^{4} \nabla_i^2 \left(I_i c_i \nabla_i + C_i s_i \Delta_i\right) w, \\ Q_1 &= -\sum_{i=1}^{4} \nabla_i^2 \left(I_i s_i \nabla_i + C_i c_i \Delta_i\right) w, \end{aligned}\right\} \quad (8)$$

By virtue of (7), equation (5) takes the form

$$\sum_{i=1}^{4} \nabla_i^2 \left(I_i \nabla_i^2 + C_i S_i^2\right) w - z = 0 \quad (9)$$

Equation (5) is solved under the condition

$$M_n \delta \frac{\partial W}{\partial n}\bigg| = 0, R_n \delta W|_G = 0 \quad (10)$$

where n – outer normal direction; (□-variation operation sign; $M_n -$ normal bending moment,

$$\begin{gathered} M_n = M_1 \cos^2 \alpha + \\ +(H_1 + H_2) \cos \alpha \sin \alpha + M_2 \sin 2\alpha \end{gathered} \quad (11)$$

$R_n -$ normal ground reaction, having the form

$$\begin{gathered} R_n = Q_n + \frac{\partial M_n \tau}{\partial S}, \\ Q_n = Q_1 \cos \alpha + Q_2 \sin \alpha, \\ M_n = H_1(\cos^2 \alpha - \sin^2 \alpha) + \\ +(M_2 - M_1) \cos \alpha \sin \alpha = \\ = H_1 \cos 2\alpha + \frac{1}{2}(M_2 - M_1) \sin 2\alpha, \end{gathered} \quad (12)$$

here (□-angle between outer normal and axis x; W – plate deflection; W – plate deflection; S – the length of the arc for the boundary of the Γ region of the plate

For the purpose of computing mesh plates with complex shapes, this section presents a computational technique that builds a joint combination of R-function approaches by V.L. Rvachev and Bubnov Galerkin. The procedure consists of two steps:

Constructing coordinate sequences, also known as solution structures;

- building equations for solving problems (discretization by spatial variables) using the Bubnov-Galerkin method;
- approximating double integral values;
– solving equations for solving problems;
- determining necessary parameter values;
– registering calculation outcomes.

Depending on how the mesh plate's edges are secured, differentiable equations (1.3.5) for the mesh plate's equilibrium can be solved given suitable boundary conditions based on the number of families of rods. Condition (10), in a generalized form, determines these boundary conditions.

Generally speaking, we use the form to represent the coordinate sequences that match criterion (5).

$$W = \sum_{i=1}^{n} C_i \phi_i \quad (13)$$

where C_i – unknown coefficients to be determined; $\varphi_i -$ basis system of coordinate functions, meeting the boundary criteria, which were created using V.L. Rvachev's R-function approach.

If we replace (13) with (5), we get the following system of algebraic equations by applying the Bubnov-Galerkin techniques;

$$AC=f \quad (14)$$

Where

$$\begin{gathered} A = \{a_{ij}\}, dim(A) = n \times n, \\ C = \{c_i\}, f = \{f_i\}, dim(C) = \\ = n \times 1, dim(f) = n \times 1, \end{gathered}$$

Here in general elements a_{ij} and f_i for equations (5) have a look

$$a_{ij} = \iint_{\Omega} \tilde{A} W_i W_j d\Omega \quad (15)$$

$$f_j = \iint_{\Omega} Z W_j d\Omega \quad (16)$$

Here

$$\begin{aligned}\tilde{A}W_i := \quad & (D_{11}+K_{11})\frac{\partial^4 W}{\partial x^4} + 2(D_{61}+D_{16})\frac{\partial^4 W}{\partial x^3\partial y} + (3D_{66}+K_{16})\frac{\partial^4 W}{\partial x^2\partial y^2} \\ & +2(D_{62}+K_{16})\frac{\partial^4 W}{\partial x\partial y^3} + (D_{22}+K_{11})\frac{\partial^4 W}{\partial y^4} \\ & +[2\frac{\partial}{\partial x}(D_{11}+K_{11}) - 2\frac{\partial}{\partial y}(2D_{61}-K_{16})]\frac{\partial^3 W}{\partial x^3} \\ & +[3\frac{\partial}{\partial x}(2D_{61}-K_{16}) + \frac{\partial}{\partial y}(3D_{66}-K_{16}^0)]\frac{\partial^3 W}{\partial x^2\partial y} \\ & +[3\frac{\partial}{\partial y}(2D_{62}+K_{16}) + \frac{\partial}{\partial x}(2D_{62}+K_{16})]\frac{\partial^3 W}{\partial x\partial y^2} \\ & +[\frac{\partial}{\partial y}(D_{22}+K_{11})]\frac{\partial^3 W}{\partial y^3} \\ & +[\frac{\partial^2}{\partial x^2}(D_{11}+K_{11}) + \frac{\partial^2}{\partial x\partial y}(2D_{61}-K_{16}) + \frac{\partial^2}{\partial y^2}(2D_{61}+K_{16}) \\ & +\frac{\partial^2}{\partial x\partial y}(D_{66}+K_{16}^0) + \frac{\partial^2}{\partial y^2}(2D_{62}+K_{16})]\frac{\partial^2 W}{\partial x^2} \\ & +[\frac{\partial^2}{\partial x^2}(D_{12}-K_{11}) + \frac{\partial^2}{\partial x\partial y}(2D_{62}+K_{16}) + \frac{\partial^2}{\partial y^2}(D_{22}+K_{11})]\frac{\partial^2 W}{\partial y^2}\end{aligned} \tag{17}$$

Equations (14) and (15) respectively assume the following form if we examine specific instances of equations (5).

$$a_{ij} = \iint_\Omega (D_1\frac{\partial^4 W_i}{\partial x^4} + D_3\frac{\partial^4 W_i}{\partial x^2\partial y^2} + D_2\frac{\partial^4 W_i}{\partial y^4})_i W_j d\Omega \tag{18}$$

$$f_j = \iint_\Omega a W_j d\Omega \tag{19}$$

Here a, depending on the consideration of equations (15) or special cases, respectively, takes the following values:

$$\text{a:=}\frac{Z}{I}\text{;a:=}\frac{a_3}{EJ_1}Z\text{ ; a:=}\frac{a_4}{EJ_1}Z\text{ ;}$$

In this case, the expressions for a_{ij} in equations (15) or others similar in form to expressions (18), difference is, expressions here D_1, D_2, D_3 will differ.

Equation (16) is solved by the Gaussian elimination method. Unknown coefficients C_i are determined. Then substituting the values C_i into (13), we will find solution W. Afterwards, the transverse forces of the plate and the values of its bending and torque moments are calculated using the structural formulas. Further, using the known values of bending (M_1, M_2) and torque (M_{12}) moments and shear forces Q_1, Q_2; force values are calculated (N_i^*, S_i^*) and moments M_i^* rod.

We will see that the precise Gauss formula is used to determine the value of double integrals.

Let's now examine how structural formulas are actually constructed. Anisotropic plate structural formulas are utilized for this purpose; the lone exception being when the coefficient D_{ij} is changed into D_{ij}^* in case of mesh plates.

when the edges of the plate are rigidly clamped, the structure of the solution has the form [4-5]:

$$W = \omega^2 \Phi \tag{20}$$

The simply supported boundary condition is now examined. In this instance, the solution's structure takes the form

$$W(x) = \omega\Phi_1 - \frac{\omega^2}{2(A_1-\omega)}\left(A_1(2D_1\Phi_1 + \Phi_1 D_2\omega) + 2A_2T_1\Phi_1 - \frac{1}{\rho}A_3\Phi_1\right) + \omega^3\Phi_2 \tag{21}$$

We will look through the shifted boundary condition. For example, when Γ_1 – area Γ_1 – rigidly clamped plate, and the rest $\Gamma_1 = \Gamma - \Gamma_1$ free, hen the solution structure according to [4-5] has the form

$$W \;=\; \omega_1^2\,\Phi_1 + \frac{\omega_1^2\,\omega_2^2}{2\left(\omega_1^2+\omega_2^2\right)}\times$$
$$\times\{\omega_2^2\,\Phi_2 - \{\frac{\omega_2}{3}\,(S_1^{*(2)}[1-\frac{\omega_2^2}{2S_1^*}\,B_2^{*(2)}] + B_3^{*(2)} - \frac{2A_6^*}{S_1^*}\,KB_2^{*(2)})\} - \frac{1}{S_2}\,B_2^{*(2)}\}\,(W_1^2\,\Phi_1) \tag{22}$$

here Φ_1, Φ_2 – undefined components of structural formulas, which are usually presented in the form [4-5]:

$$\Phi_s = \sum_{i=1}^{m_s} C_{is}\psi_i^s,\ \text{s=1,2;} \tag{23}$$

where $\{\psi_i^s\}$ – selected sequence of polynomials, such as power polynomial, Chebyshev polynomial, trigonometric polynomial, etc.; ω_1, ω_2 – correspondingly normalized boundary equations Γ_1 and Γ_2 areas Γ_1 and Γ_2.

4. Discussion

Calculate mesh plates with a rhomboidal form is the focus of this section. Here the plate arrangements with their complex shapes are considered [7–10].

Assuming that the edges of the hexagonal plate (Fig. 1) are simply supported and firmly clamped, we can compute the surface area of the plate.

The problem comes down to integrating equation (15), i.e.

$$D_1\frac{\partial^4 W}{\partial x^4} + D_3\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_2\frac{\partial^4 W}{\partial y^4} = \frac{z}{I} \tag{24}$$

in case

$$W|\Gamma = 0, M_4|\Gamma = 0 \tag{25}$$

The geometry equation of the region in this case has the form:

$$\omega = \omega_1 \wedge \omega_2 \wedge_0 \omega_3 \tag{26}$$
$$\omega_1 = \frac{\left(\frac{3}{4} - y^2\right)}{\sqrt{3}},$$
$$\omega_2 = f_1 \wedge_0 f_3, \omega_3 =$$
$$= f_4 \wedge_0 f_5, f_1 = \frac{\left(\sqrt{3} - \sqrt{3}x - y\right)}{2},$$
$$f_3 = \left(\sqrt{3} + \sqrt{3}x - y\right)\sqrt{2},$$
$$f_4 = \frac{\frac{\sqrt{3}}{2}\left(\sqrt{3} + \sqrt{3}x + y\right)}{2},$$
$$f_5 = \left(\sqrt{3} - \sqrt{3}x + y\right)/2$$

Take note that the formulas for bending moments for traditional orthotropic elastic plates differ slightly from the expressions for moments in form used here. Consequently, we describe the moments of mesh plates in standard form and create structural formulas for expressing them.

For this purpose, consider the relation:

$$\begin{gathered} M_1 = 2c^2\left[(Ic^2 + Cs^2)\frac{\partial^2 w}{\partial x^2} + (I-C)s^2\frac{\partial^2 w}{\partial y^2}\right], \\ M_2 = 2s^2\left[(I-C)c^2\frac{\partial^2 w}{\partial x^2} + (Is^2 + Cc^2)\frac{\partial^2 w}{\partial y^2}\right], \\ H_1 = -2c^2(2Is^2 + C\cos 2\phi)\frac{\partial^2 w}{\partial x \partial y}, \\ H_2 = 2s^2(C\cos 2\phi - 2Ic^2)\frac{\partial^2 w}{\partial x \partial y} \end{gathered} \tag{27}$$

We will rewrite the relation (27) in form

$$M_1 = D_{1c}\left[\frac{\partial^2 w}{\partial x^2} + v_{2c}\frac{\partial^2 w}{\partial y^2}\right],$$
$$M_2 = D_{2c}\left[\frac{\partial^2 w}{\partial y^2} + v_{1c}\frac{\partial^2 w}{\partial x^2}\right], \qquad (28)$$
$$H_1 = \alpha_1 \frac{\partial^2 w}{\partial x \partial y},$$
$$H_2 = \alpha_2 \frac{\partial^2 w}{\partial x \partial y}.$$

where

$$D_{1c} = 2c^2(Ic^2 + Cs^2),$$

$$v_{2c} = \frac{(I - C)s^2}{Ic^2 + Cs^2},$$

$$D_{2c} = 2s^2(Is^2 + Cc^2),$$

$$v_{1c} = \frac{(I - C)c^2}{Is^2 + Cc^2},$$

$$C = GJ_3/a, I = EJ_1/a,$$

$$C = \cos\phi, S = \sin\phi,$$

$$\alpha_1 = -2c^2(2Is^2 + C\cos 2\phi)\frac{\partial^2 w}{\partial x \partial y},$$

$$\alpha_2 = 2s^2(C\cos 2\phi - 2Ic^2)\frac{\partial^2 w}{\partial x \partial y}$$

Rhomboidal plate structure $\varphi = \frac{\pi}{4}$. Thus s=c и $D_{1c} = D_{2c}$; $v_{1c} = v_{2c}$, therefore, we consider and $J_3 = 0$ we can rewrite these formulas in the form

$$M_1 = D_c\left(\frac{\partial^2 w}{\partial x^2} + v_c\frac{\partial^2 w}{\partial y^2}\right),$$

$$M_2 = D_c\left(\frac{\partial^2 w}{\partial y^2} + v_c\frac{\partial^2 w}{\partial x^2}\right), \qquad (29)$$

$$H_1 = H_2 = -4Ic^2s^2$$

where $D_C = D_C = D_{2C}, v_c = v_{1c} = v_{2c}$.

According to (29) we will have this

$$M_n = D_c\left(\frac{\partial^2 w}{\partial n^2} + v_c\frac{\partial^2 w}{\partial \tau^2}\right) \qquad (30)$$

so

$M_n = M_1\cos^2\alpha + M_2\sin^2\alpha + 2M_{12}\sin\alpha\cos\alpha$.

Here n – outer normal, □ – tangent.

Considering the remarks above, relations of the form determine the boundary condition of a simply supported plate with a rhomboidal structure.

$$W|r = 0, \left(\frac{\partial^2 w}{\partial n^2} + v_c\frac{\partial^2 w}{\partial \tau^2}\right)\Big|_{\&_r} = 0 \qquad (31)$$

Consequently, the structure of this boundary condition's solution in this instance is shown as

$$W = \omega\Phi -$$
$$-\frac{\bar{\omega}^2}{2}[\Phi(D_2\bar{\omega} + v_c T_2\bar{\omega}) + 2D_1\Phi] \qquad (32)$$

We note that if the stiffness of the rods is not taken into account $J_3 = 0$, then C=0, ν_c =1 and (3.2.7) takes the form

$$W = \omega\Phi -$$
$$= \frac{\bar{\omega}^2}{2}[\Phi(D_2\bar{\omega} + T_2\bar{\omega}) + 2D_1\Phi] \qquad (33)$$

Let us now examine the rhombic structure's hexagonal plate, as depicted in Figure 1. Let a firmly clamped rhomboidal mesh plate form the image's border in Figure 1. Then, the structure of the answer to this issue takes the following form:

$$W = \omega^2\Phi \qquad (34)$$

Here ω is found out with the formula (26) and Φ is given as (23).

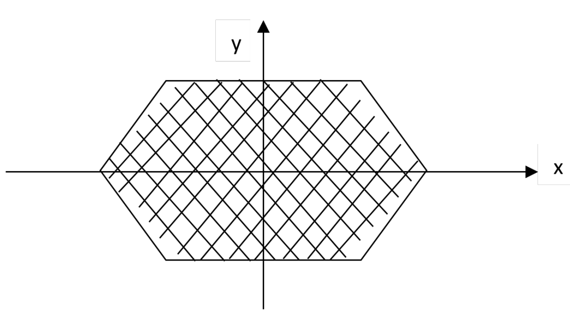

Figure 1 – Complex poligon

x	W n=3 c=20
-1	0
-0,9	0,00010152
-0,8	0,00071714
-0,7	0,00205652
-0,6	0,00400341
-0,5	0,00626503
-0,4	0,00851907
-0,3	0,01049807
-0,2	0,01201645
-0,1	0,01296362
0	0,01328472
0,1	0,01296362
0,2	0,01201645
0,3	0,01049807
0,4	0,00851907
0,5	0,00626503
0,6	0,00400341
0,7	0,00205652
0,8	0,00071714
0,9	0,00010152
1	0

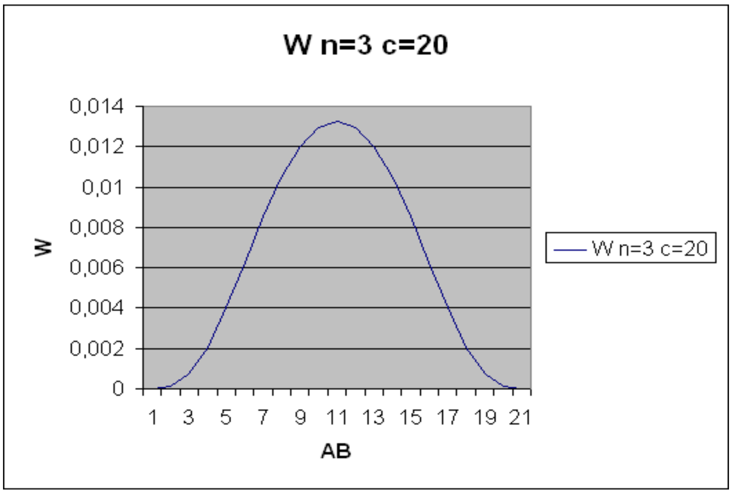

Figure 2 – The figure shows numerical and graphical results for one of the sections1.

Below (Fig. 2) the computational experiment's findings are displayed.

5. Conclusions

Summarize the key findings and their significance. Clearly state the main conclusions drawn from the study and their implications. Avoid introducing new data or extensive discussions not previously covered.

Author Contributions

Conceptualization, F.N.; Methodology, F.N. and B.S.; Software, N.K.; Formal Analysis, F.N., B.S. and N.K.; Investigation, B.S.; Resources, F.N., B.S. and N.K.; Data Curation, B.S. and N.K.; Writing – Original Draft Preparation, F.N. and B.S.; Writing – Review & Editing, F.N.; Visualization, B.S. and N.K.; Supervision, F.N.

Conflicts of Interest

The authors declare no conflict of interest.

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