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NUMERICAL MODELING OF THE FILTRATION PROCESS IN DYNAMICALLY INTERCONNECTED MULTILAYER GAS FIELDS

Abstract. This article is devoted to modeling of the gas filtration process in a dynamically interconnected multilayer porous medium. The article is devoted to modeling the process of gas filtration in a dynamically interconnected multilayer porous medium. In the article, the process of gas filtration in a heterogeneous three layer porous medium with low-permeability intermediate layers and the dynamic interaction between the layers are described by a mathematical model based on a system of differential equations of parabolic type. This mathematical model is numerically simulated using finite difference methods, i.e. explicit and implicit schemes. Since the resulting system of finite difference equations is nonlinear with respect to the pressure function, a quasilinear method was used. The dynamics of the pressure function over time was analyzed for time intervals of 360, 720 and 1080 days, and during this period the pressure distribution in the layers, the rate of pressure drop around the well and the dynamics of interlayer interaction were studied. The calculation results are presented in numerical and graphical form, which accurately reflect how the interlayer movement of the gas flow occurs. Using graphical analysis, the time step limit was determined, ensuring the stability of the computational process in the explicit scheme: a stable calculation is carried out only with a dimensionless time step $\Delta t \leq 1.7e\text{-}4$. Also, the calculations carried out using the implicit scheme showed that this method has more stable stability compared to the explicit scheme. The results show that with a large permeability coefficient of the formation, the pressure distribution accelerates, and in the wells the pressure drop slows down. At the same time, in directly connected multilayer porous media, the permeability coefficient of the layers plays an important role. Based on the obtained results, it is possible to carry out calculations for various parameters to improve the efficiency of gas field development. It is also possible to analyze and forecast oil and gas deposits using software created on the basis of numerical models and algorithms developed in the article.

Key words: gas, filtration, multilayer, porous medium, parabolic equations, numerical modeling, finite difference method, explicit scheme, implicit scheme, quasilinear method, pressure, stability condition, conductivity coefficient, software, mathematical model.

1 Introduction and preliminaries

The work is devoted to modeling the process of liquid filtration in oil and gas fields, and covers the fundamentals of solving differential equations describing multiphase flows using numerical methods. Recommendations are given on mathematical modeling of reservoir systems, creation of computer models and their analysis, which can be used not only in modeling oil, but also underground water and gas fields [1,2].

The article is devoted to the calculation of the boundary value problem of the gas filtration process in three layer interconnected porous media. The results of computational experiments show that with a small parameter of a low-permeability layer, the pressure drop occurs mainly in the middle layer [3]. The developed mathematical model, computational algorithm and software can be effectively applied in the analysis and forecasting of filtration processes in multi-layer oil and gas fields [4-8].

The article develops a mathematical model of the two-phase filtration process in layers of porous media. Underground studies of the process of two-phase non-stationary filtration in interconnected two-layer porous media were also analyzed. Based on the research results, a numerical solution and a boundary value problem were developed. Accurate and effective methods were used to solve two-dimensional filtration problems [9-12].

The presented analysis shows that when determining and analyzing the indicators of oil and gas field development, it is necessary to develop effective numerical methods, computational algorithms and their software. In particular, in-depth study and modeling of the gas filtration process in a heterogeneous three layer porous medium with low-permeability intermediate layers, taking into account the dynamic connections between the layers, is of great importance when developing multilayer gas fields.

2 Mathematical model

Due to the low permeability of the intermediate layers, the movement of liquid in them occurs only in the vertical direction. Therefore, the mathematical model of such a process is the following system of equations, consisting of three interconnected one-dimensional differential equations of parabolic type.

$$\begin{cases} \frac{\partial}{\partial x}\left[K_1 h_1 \frac{\partial P_1^2}{\partial x}\right] = 2a\mu m h_1 \frac{\partial P_1}{\partial t} - \frac{K_{\Pi 1}}{h_{\Pi 1}}(P_2^2 - P_1^2) \\ \frac{\partial}{\partial x}\left[K_2 h_2 \frac{\partial P_2^2}{\partial x}\right] = 2a\mu m h_2 \frac{\partial P_2}{\partial t} + \frac{K_{\Pi 1}}{h_{\Pi 1}}(P_2^2 - P_1^2) - \frac{K_{\Pi 2}}{h_{\Pi 2}}(P_3^2 - P_2^2) - Q \\ \frac{\partial}{\partial x}\left[K_3 h_3 \frac{\partial P_3^2}{\partial x}\right] = 2a\mu m h_3 \frac{\partial P_3}{\partial t} + \frac{K_{\Pi 2}}{h_{\Pi 2}}(P_3^2 - P_2^2). \end{cases} \quad (1)$$
$$0 < x < L, t > 0.$$

The initial and boundary conditions of the problem:

$$P_1(x, t_0) = P_{1H}(x), P_2(x, t_0) = P_{2H}(x), P_3(x, t_0) = P_{3H}(x); \quad (2)$$

$$\begin{cases} -K_1 h_1 \frac{\partial P_1}{\partial \mathrm{x}} |\mathrm{x} = 0 = \alpha(P_A - P_1); \\ K_1 h_1 \frac{\partial P_1}{\partial x} |x = L = \alpha(P_A - P_1). \end{cases} \quad (3)$$

$$\begin{cases} -K_2 h_2 \frac{\partial P_2}{\partial x} |x = 0 = \alpha(P_A - P_2); \\ K_2 h_2 \frac{\partial P_2}{\partial x} |x = L = \alpha(P_A - P_2). \end{cases} \quad (4)$$

$$\begin{cases} -K_3 h_3 \frac{\partial P_3}{\partial x} |x = 0 = \alpha(P_A - P_3); \\ K_3 h_3 \frac{\partial P_3}{\partial x} |x = L = \alpha(P_A - P_3). \end{cases} \quad (5)$$

$$Q = \sum_{i_q=1}^{N_q} q_{i_q} \delta\left(x - x_{i_q}\right); i_q = 1, \ldots N_q; \quad (6)$$

Here: $\delta -$ Dirac delta function; $n -$ normal in the border area; $\mu -$ gas viscosity; P_A −boundary pressure; $P_1, P_2, P_3 -$ pressure in layers; $K_1, K_2, K_3 -$ coefficient of permeability of layers; $P_{1H}, P_{2H}, P_{3H} -$ initial pressure of the layers; $K_{\Pi 1}, K_{\Pi 2} -$ weak permeability layer coefficients; $m-$ porous coefficient of layers; $h_1, h_2, h_3 -$ layers thickness; $q_{i_q} - i_q -$ well flow rate; a – gas saturation; L – layer length; N_q – number of wells; $\alpha = \begin{cases} 0, & \text{– fixed at the boundary;} \\ 1, & \text{– not fixed at the boundary.} \end{cases}$

To numerically solve the boundary value problem, we will reduce it to a dimensionless form [13,14]. For this purpose, we will introduce the following notations into the system of equations (1):

$$x^* = \frac{x}{L}; \tau = \frac{K_x t P_x}{2am\mu L^2}; K_1^* = \frac{K_1}{K_x}; K_2^* = \frac{K_2}{K_x};$$

$$K_3^* = \frac{K_3}{K_x}; K_{\Pi 1}^* = \frac{K_{\Pi 1}}{K_x}; K_{\Pi 2}^* = \frac{K_{\Pi 2}}{K_x};$$

$$h_1^* = \frac{h_1}{h_x};\ h_2^* = \frac{h_2}{h_x};\ h_3^* = \frac{h_3}{h_x};\ h_{\Pi 1}^* = \frac{h_{\Pi 1}}{h_x};$$

$$h_{\Pi 2}^* = \frac{h_{\Pi 2}}{h_x};\ P_1^* = \frac{P_1}{P_x};\ P_2^* = \frac{P_2}{P_x};$$

$$P_3^* = \frac{P_3}{P_x};\ q^* = \frac{\mu L^2}{K_x h_x P_x} Q, \alpha^* = \alpha \frac{L}{K_x h_x}.$$

In this case, K_x, h_x, P_x are the values of the permeability, thickness, and pressure of the layer at a certain value, respectively. α^*- dimensional parameter.

By performing these transformations in the system of equations and omitting the symbol “*” in the equation, we obtain at the following dimensionless system of equations:

$$\begin{cases} \frac{\partial}{\partial x}\left[K_1 h_1 \frac{\partial P_1^2}{\partial x}\right] = \frac{h_1 \partial P_1}{\partial \tau} - \frac{K_{\Pi 1} L^2}{h_{\Pi 1} h_x^2}(P_2^2 - P_1^2), \\ \frac{\partial}{\partial x}\left[K_2 h_2 \frac{\partial P_2^2}{\partial x}\right] = \frac{h_2 \partial P_2}{\partial \tau} + \frac{K_{\Pi 1} L^2}{h_{\Pi 1} h_x^2}(P_2^2 - P_1^2) - \frac{K_{\Pi 2} L^2}{h_{\Pi 2} h_x^2}(P_3^2 - P_2^2) - q, \\ \frac{\partial}{\partial x}\left[K_3 h_3 \frac{\partial P_3^2}{\partial x}\right] = \frac{h_3 \partial P_3}{\partial \tau} + \frac{K_{\Pi 2} L^2}{h_{\Pi 2} h_x^2}(P_3^2 - P_2^2). \end{cases} \tag{7}$$

$$0 < x < 1; \tau > 0.$$

The initial and boundary conditions are as follows:

$$P_1(x, t_0) = P_{1H}(x), P_2(x, t_0) = P_{2H}(x), P_3(x, t_0) = P_{3H}(x); \tag{8}$$

$$\begin{cases} -K_1 h_1 \frac{\partial P_1}{\partial \mathrm{x}} |\mathrm{x} = 0 = \alpha(P_A - P_1); \\ K_1 h_1 \frac{\partial P_1}{\partial x} |x = 1 = \alpha(P_A - P_1). \end{cases} \tag{9}$$

$$\begin{cases} -K_2 h_2 \frac{\partial P_2}{\partial x} |x = 0 = \alpha(P_A - P_2); \\ K_2 h_2 \frac{\partial P_2}{\partial x} |x = 1 = \alpha(P_A - P_2). \end{cases} \tag{10}$$

$$\begin{cases} -K_3 h_3 \frac{\partial P_3}{\partial x} |x = 0 = \alpha(P_A - P_3); \\ K_3 h_3 \frac{\partial P_3}{\partial x} |x = 1 = \alpha(P_A - P_3). \end{cases} \tag{11}$$

For convenience, the following notations we defined in the system of equations:

$$T_1 = K_1 h_1,\ T_2 = K_2 h_2,\ T_3 = K_3 h_3,$$

$$R_1 = \frac{K_{\Pi 1} \mathrm{L}^2}{h_{\Pi 1} h_x^2},\ \ R_2 = \frac{K_{\Pi 2} L^2}{h_{\Pi 2} h_x^2}.$$

We will solve the dimensionless boundary value problem (7) – (11) using the finite difference method.

Numerical modeling. To solve the dimensionless boundary value problem (7)-(11), we will construct its numerical model, using the finite difference method. To do this, we construct the following grid on the x-axis $0 \le x \le 1$ and $0 < t < T_0$ over time intervals:

$$\omega_{x,\tau} = \{x_i = i\Delta x; i = \overline{1,n}; \tau_l = l\Delta\tau;$$

$$l = 0,1,2,\dots,N_\tau; \Delta\tau = \frac{T}{N_\tau}\}.$$

Here Δx – step along the x-axis; $\Delta\tau$ – step in time; $\omega_{x,\tau}$ −grid area.

It is known that one of the methods for discretizing a system of equations of parabolic type is the finite difference method. Applying this method to the boundary value problem, we reduce the system of differential equations (7) – (11) to the following system of finite difference equations with an explicit scheme:

$$\frac{T_{1i-0,5}P_{1i-1}^{2(l)} - (T_{1i-0,5} + T_{1i+0,5})P_{1i}^{2(l)} + T_{1i+0,5}P_{1i+1}^{2(l)}}{\Delta x^2} = h_{1i}\frac{P_{1i}^{(l+1)} - P_{1i}^{(l)}}{\Delta\tau} - R_1P_{2i}^{2(l)} + R_1P_{1i}^{2(l)},$$

$$\frac{T_{2i-0,5}P_{2i-1}^{2(l)} - (T_{2i-0.5} + T_{2i+0,5})P_{2i}^{2(l)} + T_{2i+0,5}P_{2i+1}^{2(l)}}{\Delta x^2} =$$

$$= h_{2i}\frac{P_{2i}^{(l+1)} - P_{2i}^{(l)}}{\Delta\tau} + R_1P_{2i}^{2(l)} - R_1P_{1i}^{2(l)} - R_2P_{3i}^{2(l)} + R_2P_{2i}^{2(l)} - \delta_i q_i,$$

$$\frac{T_{3i-0,5}P_{3i-1}^{2(l)} - (T_{3i-0.5} + T_{3i+0,5})P_{3i}^{2(l)}{}_{3i+0,5}P_{3i+1}^{2(l)}}{\Delta x^2} = h_{3i}\frac{P_{3i}^{(l+1)} - P_{3i}^{(l)}}{\Delta\tau} + R_2P_{3i}^{2(l)} - R_2P_{2i}^{2(l)},$$

$$i = 1,2,\dots n-1.$$

Since the resulting system of finite difference equations is nonlinear with respect to the pressure function, its direct solution is quite complicated, so we apply the quasilinear method to the equations [15-20]. According to the quasilinear method, the nonlinear parts of the equation can be written as follows:

$$\phi(P) \cong \phi(\tilde{P}) + (P - \tilde{P})\frac{\partial\phi(\tilde{P})}{\partial P} \qquad (12)$$

In this case, the function $\tilde{P}$ is an approximate value of P. If we write formula (12) with respect to the pressure function, then we obtain the following formulas:

$$P_{1i}^{2(l)} \cong 2\tilde{P}_{1i}^{(l)}P_{1i}^{(l)} - \tilde{P}_{1i}^{2(l)};$$

$$P_{2i}^{2(l)} \cong 2\tilde{P}_{2i}^{(l)}P_{2i}^{(l)} - \tilde{P}_{2i}^{2(l)};$$

$$P_{3i}^{2(l)} \cong 2\tilde{P}_{3i}^{(l)}P_{3i}^{(l)} - \tilde{P}_{3i}^{2(l)}.$$

$$\frac{T_{1i-0,5}(2\tilde{P}^{(l)}{}_{1i-1}P_{1i-1}^{(l)} - \tilde{P}_{1i-1}^{2(l)}) - (T_{1i-0,5} + T_{1i+0,5})(2\tilde{P}^{(l)}{}_{1i}P_{1i}^{(l)} - \tilde{P}_{1i}^{2(l)}) + T_{1i+0,5}(2\tilde{P}^{(l)}{}_{1i+1}P_{1i+1}^{(l)} - \tilde{P}_{1i+1}^{2(l)})}{\Delta x^2} =$$

$$= h_{1i}\frac{P_{1i}^{(l+1)} - P_{1i}^{(l)}}{\Delta\tau} - R_1(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)} - \tilde{P}_{2i}^{2(l)}) + R_1(2\tilde{P}^{(l)}{}_{1i}P_{1i}^{(l)} - \tilde{P}_{1i}^{2(l)}) + R_3(2\tilde{P}^{(l)}{}_{3i}P_{3i}^{(l)} - \tilde{P}_{3i}^{2(l)}),$$

$$\frac{T_{2i-0,5}(2\tilde{P}^{(l)}{}_{2i-1}P_{2i-1}^{(l)} - \tilde{P}_{2i-1}^{2(l)}) - (T_{2i-0.5} + T_{2i+0,5})(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)} - \tilde{P}_{2i}^{2(l)}) + T_{2i+0,5}(2\tilde{P}^{(l)}{}_{2i+1}P_{2i+1}^{(l)} - \tilde{P}_{2i+1}^{2(l)})}{\Delta x^2} =$$

$$= h_{2i}\frac{P_{2i}^{(l+1)} - P_{2i}^{(l)}}{\Delta\tau} + R_1(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)} - \tilde{P}_{2i}^{2(l)}) - R_1(2\tilde{P}^{(l)}{}_{1i}P_{1i}^{(l)} - P_{1i}^{2(l)}) - R_2(2\tilde{P}^{(l)}{}_{3i}P_{3i}^{(l)} - \tilde{P}_{3i}^{2(l)}) +$$

$$+R_2(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)} - \tilde{P}_{2i}^{2(l)}) - \delta_i q_i,$$

$$\frac{T_{3i-0,5}(2\tilde{P}^{(l)}{}_{3i-1}P_{3i-1}^{(l)}-\tilde{P}_{3i-1}^{2(l)})-\left(T_{3i-0.5}+T_{3i+0,5}\right)(2\tilde{P}^{(l)}{}_{3i}P_{3i}^{(l)}-\tilde{P}_{3i}^{2(l)})+T_{3i+0,5}(2\tilde{P}^{(l)}{}_{3i+1}P_{3i+1}^{(l)}-\tilde{P}_{3i+1}^{2(l)})}{\Delta x^2}=$$

$$=h_{3i}\frac{P_{3i}^{(l+1)}-P_{3i}^{(l)}}{\Delta\tau}+R_2(2\tilde{P}^{(l)}{}_{3i}P_{3i}^{(l)}-\tilde{P}_{3i}^{2(l)})-R_2(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)}-\tilde{P}_{2i}^{2(l)})+R_3(2\tilde{P}^{(l)}{}_{1i}P_{1i}^{(l)}-\tilde{P}_{1i}^{2(l)}),$$

$$i=1,2,\dots n-1.$$

From this system of equations, we find the pressure functions $P_{1i}^{l+1}, P_{2i}^{l+1}, P_{3i}^{l+1}$ as follows:

$$P_{1i}^{l+1}=\Big(T_{1i-0,5}(2\tilde{P}^{(l)}{}_{1i-1}P_{1i-1}^{(l)}-\tilde{P}_{1i-1}^{2(l)})-\left(T_{1i-0,5}+T_{1i+0,5}\right)(2\tilde{P}^{(l)}{}_{1i}P_{1i}^{(l)}-\tilde{P}_{1i}^{2(l)})$$
$$++T_{1i+0,5}(2\tilde{P}^{(l)}{}_{1i+1}P_{1i+1}^{(l)}-\tilde{P}_{1i+1}^{2(l)})\Big)*$$

$$*\frac{\Delta\tau}{h_{1i}\Delta x^2}-h_{1i}\frac{P_{1i}^{l}}{\Delta\tau}-\frac{\Delta\tau}{h_{1i}}\Big(R_1(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)}-\tilde{P}_{2i}^{2(l)})+R_1(2\tilde{P}^{(l)}{}_{1i}P_{1i}^{(l)}-\tilde{P}_{1i}^{2(l)})+R_3(2\tilde{P}^{(l)}{}_{3i}P_{3i}^{(l)}-\tilde{P}_{3i}^{2(l)})\Big),$$

$$P_{2i}^{l+1}=\Big(T_{2i-0,5}(2\tilde{P}^{(l)}{}_{2i-1}P_{2i-1}^{(l)}-\tilde{P}_{2i-1}^{2(l)})-\left(T_{2i-0.5}+T_{2i+0,5}\right)(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)}-\tilde{P}_{2i}^{2(l)})$$
$$+T_{2i+0,5}(2\tilde{P}^{(l)}{}_{2i+1}P_{2i+1}^{(l)}-\tilde{P}_{2i+1}^{2(l)})\Big)*$$

$$\frac{\Delta\tau}{h_{2i}\Delta x^2}-h_{2i}\frac{P_{2i}^{l}}{\Delta\tau}+\frac{\Delta\tau}{h_{2i}}\Big(R_1(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)}-\tilde{P}_{2i}^{2(l)})-R_1(2\tilde{P}^{(l)}{}_{1i}P_{1i}^{(l)}-P_{1i}^{2(l)})-R_2(2\tilde{P}^{(l)}{}_{3i}P_{3i}^{(l)}-\tilde{P}_{3i}^{2(l)})+$$
$$+R_2(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)}-\tilde{P}_{2i}^{2(l)})-\delta_i q_i\Big),$$

$$P_{3i}^{l+1}=\Big(T_{3i-0,5}(2\tilde{P}^{(l)}{}_{3i-1}P_{3i-1}^{(l)}-\tilde{P}_{3i-1}^{2(l)})-\left(T_{3i-0.5}+T_{3i+0,5}\right)(2\tilde{P}^{(l)}{}_{3i}P_{3i}^{(l)}-\tilde{P}_{3i}^{2(l)})$$
$$+T_{3i+0,5}(2\tilde{P}^{(l)}{}_{3i+1}P_{3i+1}^{(l)}-\tilde{P}_{3i+1}^{2(l)})\Big)*$$

$$*\frac{\Delta\tau}{h_{3i}\Delta x^2}-h_{3i}\frac{P_{3i}^{l}}{\Delta\tau}+\frac{\Delta\tau}{h_{3i}}\Big(R_2(2\tilde{P}^{(l)}{}_{3i}P_{3i}^{(l)}-\tilde{P}_{3i}^{2(l)})-R_2(2\tilde{P}^{(l)}{}_{2i}P_{2i}^{(l)}-\tilde{P}_{2i}^{2(l)})+R_3(2\tilde{P}^{(l)}{}_{1i}P_{1i}^{(l)}-\tilde{P}_{1i}^{2(l)})\Big),$$

In this system of equations, we introduce the notation $P_{1i}^{(s)}=\tilde{P}_{1i}^{(l+1)}$, $P_{2i}^{(s)}=\tilde{P}_{2i}^{(l+1)}$, $P_{3i}^{(s)}=\tilde{P}_{3i}^{(l+1)}$ and continue the iteration process to the following condition:

$$\left|P_{1i}^{(s)}-P_{1i}^{(s-1)}\right|<\varepsilon,\left|P_{1i}^{(s)}-P_{1i}^{(s-1)}\right|<\varepsilon,\left|P_{3i}^{(s)}-P_{3i}^{(s-1)}\right|<\varepsilon \qquad (13)$$

To find the values of the pressure functions at the boundary point, the boundary conditions (8) – (11) can be written in the form of finite difference equations:

$$\begin{cases}(3k_{10}h_{10}-2\Delta x\alpha)P_{10}-4k_{11}h_{11}P_{11}+k_{12}h_{12}P_{12}=2\Delta x\alpha P_A;\\(3k_{1n}h_{1n}-2\Delta x\alpha)P_{1n}-4k_{1n-1}h_{1n-1}P_{1n-1}+k_{1n-2}h_{1n-2}P_{1n-2}=-2\Delta x\alpha P_A;\end{cases}$$

$$\begin{cases}(3k_{20}h_{20}-2\Delta x\alpha)P_{20}-4k_{21}h_{21}P_{21}+k_{22}h_{22}P_{22}=2\Delta x\alpha P_A;\\(3k_{2n}h_{2n}-2\Delta x\alpha)P_{2n}-4k_{2n-1}h_{2n-1}P_{2n-1}+k_{2n-2}h_{2n-2}P_{2n-2}=-2\Delta x\alpha P_A;\end{cases}$$

$$\begin{cases}(3k_{30}h_{30} - 2\Delta x\alpha)P_{30} - 4k_{31}h_{31}P_{31} + k_{32}h_{32}P_{32} = 2\Delta x\alpha P_A; \\ (3k_{3n}h_{3n} - 2\Delta x\alpha)P_{3n} - 4k_{3n-1}h_{3n-1}P_{3n-1} + k_{3n-2}h_{3n-2}P_{3n-2} = -2\Delta x\alpha P_A.\end{cases}$$

Here

$$P_{10} = \frac{4k_{11}h_{11}P_{11} + k_{12}h_{12}P_{12} + 2\Delta x\alpha P_A}{3k_{11}h_{11} - 2\Delta x\alpha}; P_{1n} = \frac{4k_{1n-1}h_{1n-1}P_{1n-1} + k_{1n-2}h_{1n-2}P_{1n-2} - 2\Delta x\alpha P_A}{3k_{1n}h_{1n} - 2\Delta x\alpha};$$

$$P_{20} = \frac{4k_{21}h_{21}P_{21} + k_{22}h_{22}P_{22} + 2\Delta x\alpha P_A}{3k_{20}h_{20} - 2\Delta x\alpha}; P_{2n} = \frac{4k_{2n-1}h_{2n-1}P_{2n-1} + k_{2n-2}h_{2n-2}P_{2n-2} - 2\Delta x\alpha P_A}{3k_{2n}h_{2n} - 2\Delta x\alpha};$$

$$P_{30} = \frac{4k_{31}h_{31}P_{31} + k_{32}h_{32}P_{32} + 2\Delta x\alpha P_A}{3k_{30}h_{30} - 2\Delta x\alpha}; P_{3n} = \frac{4k_{3n-1}h_{3n-1}P_{3n-1} + k_{3n-2}h_{3n-2}P_{3n-2} - 2\Delta x\alpha P_A}{3k_{3n}h_{3n} - 2\Delta x\alpha}.$$

Using these formulas, the values of the pressure functions at the boundary points are determined for each time interval.

3 Algorithm for solving a boundary value problem

1. Providing of initial data values:
- number of repetitions over time – *nt*;
- values of the conductivity coefficients – K_1, K_2, K_3, $K_{П1}$, $K_{П2}$;
- value of the porosity coefficient -*m*;
-layer length -*L*;
- value of the gas viscosity coefficient -*μ*;
-well flow rate -*q*;
- initial gas pressure in the layers – P_1, P_2, P_3.
2. Converting variables to dimensionless values;
3. Cycle of repetition over time $l=1...nt$;
4. Calculation of the reservoir pressure values P_{1i}, P_{2i}, P_{3i} $(i=1,n-1)$ for the time interval $l+1$.
5. Calculate the pressure values P_{10}, P_{20}, P_{30}, P_{1n}, P_{2n}, P_{3n} at the right and left boundaries.
6. The solutions found in the time interval $l+1$ will be the initial for the next step $l+2$.
7. The condition of iteration (13) is checked, if the condition is met, the next step is taken, otherwise the last found pressure values P_{1i}, P_{2i}, P_{3i} $(i=0,n)$ are given for the 4th new iteration $P_{1i}^{(s)} = P_{1i}$, $P_{2i}^{(s)} = P_{2i}$, $P_{3i}^{(s)} = P_{3i}$ and return to the 4th step.
8. Display numerical results on the screen in tabular and graphical form.
9. End of the time repetition cycle.

Computational experiments were performed and analyzed using a computer to investigate the influence of pressure variations in the upper and lower layers during gas extraction from the second layer of a three layer porous medium filtration system, dynamically interacting with low-permeability layers.

4 Computational experiments

Calculation experiments were carried out with precise initial input parameters: the length of all three layers $L_x = 10000\ m$; the initial pressure of the gas layers $P_1=300\ atm$, $P_2=300\ atm$, $P_3=300\ atm$; the gas viscosity coefficient $\mu=0.001\ sp$; the porosity coefficient $m=0.1$; the wells in the second gas layer have the same flow rate, i.e., $Q=200000\ m^3/day$. Computational experiments on the distribution of pressure in gas layers were conducted for 360, 720, and 1080 days. The results of computational experiments are presented in the form of graphs in Figures 1-4. The results of computational experiments were mainly obtained at various values of reservoir permeability coefficients and well flow rates, on the basis of which analyses were conducted.

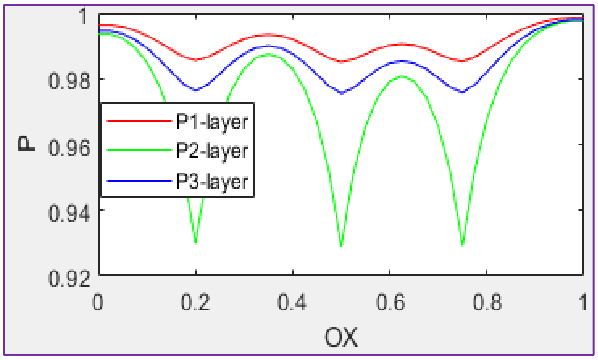

Figure 1 – Pressure distribution in layers at point n=81 ($K_{1,2,3}=0.2$; $K_{\Pi 1}=0.0001$; $K_{\Pi 2}=0.0002$; $Q_{1,2,3}=200000 m^3/day$).

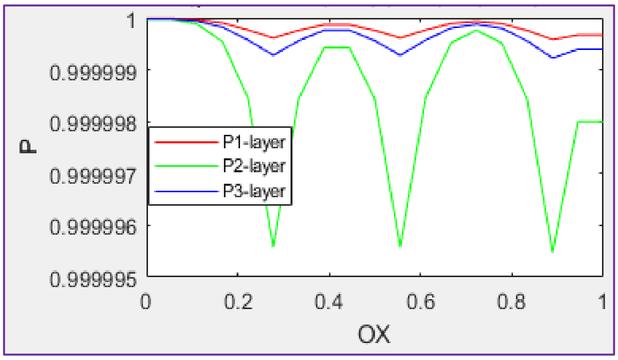

Figure 2 – Pressure distribution in layers at point n=21 ($K_{1,2,3}=0.2$; $K_{\Pi 1}=0.0001$; $K_{\Pi 2}=0.0002$; $Q_{1,2,3}=200000 m^3/day$).

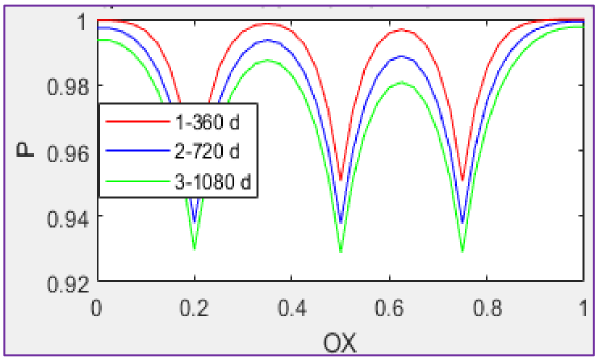

Figure 3 – Pressure distribution in the 2nd layer at point n=81 (t_1=360 day; t_2=720 day; t_3=1080 day; $K_{1,2,3}=0.1$; $K_{\Pi 1}=0.0001$; $K_{\Pi 2}=0.0002$; $Q_{1,2,3}=200000 m^3/$ day.)

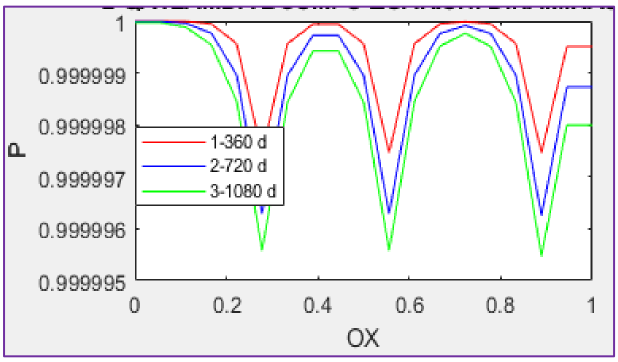

Figure 4 – Pressure distribution in the 2nd layer at point n=21 (t_1=360 day; t_2=720 day; t_3=1080 day; $K_{1,2,3}=0.1$; $K_{\Pi 1}=0.0001$; $K_{\Pi 2}=0.0002$; $Q_{1,2,3}=200000 m^3/$ day.)

Figures 1 – 4 above show the pressure drop over time in the central and adjacent wells. As can be seen from the graph, the pressure drop in the central well accelerates over time relative to the adjacent well. This is due to the influence of the adjacent well. The mathematical model was solved using two methods: implicit and explicit. Figures 1 and 3 show the results of the solution using the implicit method, while Figures 2 and 4 show the results of the solution using the explicit method. Increasing the number of points and a smaller time step help to calculate the pressure distribution more accurately and reliably. At the same time, the use of both methods is practically useful and is selected depending on the purpose of the calculation and the available computing resources.

Table 1 – Results of calculations for determining stability n=21.

Time step (days)	Time step dimensionless (Δt)	Calculation result
1 day	4.2187e-05	stable
2 days	8.4375e-05	stable
3 days	1.2656e-04	stable
4 days	1.6875e-04	stable
5 days	2.1094e-04	unstable
6 days	2.5312e-04	unstable

As can be seen from the graphs, as a result of solving the boundary value problem of the process of nonstationary gas filtration in a three layer porous medium with dynamic interaction using the explicit method, a stable state is observed in the calculation until the time step is 4 days (1.6875e-04 without dimension), and from the 5th day, a state of instability is observed.

Consequently, when solving the problem, the dimensionless time step is stable when $\Delta t \leq 1.69e\text{-}4$, and unstable when $\Delta t \geq 2.1094e\text{-}04$. From this it can be seen that the stability limit for dimensionless time step Δt can be recommended in cases of ~1.7e-4. The dimensionless time step increases in accordance with the characteristic value of the dimensionless time step and the conductivity coefficients. Therefore, this instability can also be observed with an increase in the values of the conductivity coefficients.

5 Conclusion

The analysis of all computational experiments shows that in a system with a heterogeneous three layer porous medium, the permeability coefficient of the collectors plays an important role in the gas filtration process, i.e. large values of the permeability coefficients of the main collector accelerate the pressure propagation in the collectors, and slow down the pressure drop in the wells. Similarly, increasing the values of the coefficients of the low-permeability layer accelerates the process of gas flow into the main layers. In this case, the lower the pressure in the layer, the faster the flow passes into this layer.

Conducting computer modeling and computational experiments allows us to determine the main indicators of multi-layer gas fields during their development under various reservoir parameters. The obtained numerical results are useful for analyzing the development of multi-layer gas fields with dynamic connections.

Author Contributions

Conceptualization, A. Nematov and M. M.; Methodology, A. Nematov; Software, A.B.; Formal Analysis, A. Nazirov; Writing – Original Draft Preparation, M. M.; Writing – Review & Editing, A.B.; Visualization, Zh.M.; Supervision, M.M.

Conflicts of Interest

The authors declare no conflict of interest.

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